A pesky hypothesis for the integral test and the comparison tests:

Ex 1 Does $\sum_{n=1}^{\infty} \frac{-1}{n^2+1}$ converge or diverge?

(a) Converges

(b) Diverges

Ex 2 Can you apply either comparison test to $\sum_{n=1}^{\infty} \frac{n^2-7}{n^4+5}$?

Ex 3 Can you apply the integral test or either comparison test to $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$? What are the partial sums of this series?

Def: An _____ is any series of the form

Thm: (Alternating Series Test) If a series $\sum_{n=n_0}^{\infty} a_n$ is _____ AND the sequence _____ converges _____ to _____, then the series _____.

Q. Why is this true?

Q. What if $|a_n|$ isn't monotone?

Ex 4 Show that $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$ converges.

Ex 5 Can we use the alternating series test for $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^p}$ when $p \leqslant 0$?

(a) Yes

(b) No

Q. Can we estimate the remainder of a convergent alternating series?

Ex 6 Estimate $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$ with error $< 10^{-5}$.

Def: A series $\sum_{n=1}^{\infty} a_n$ _____ if _____ converges. If $\sum_{n=1}^{\infty} a_n$ converges but $\sum_{n=1}^{\infty} |a_n|$ diverges we say $\sum_{n=1}^{\infty} a_n$ _____.

Thm: (Absolute Convergence Test) If $\sum_{n=1}^{\infty} a_n$ converges absolutely, then $\sum_{n=1}^{\infty} a_n$ converges.

Proof: a

Note: a

Ex 7 Show that $\sum_{n=1}^{\infty} \frac{\sin(n)}{n^2}$ converges.

Note: Infinite sums are _____.

- If $\sum_{n=1}^{\infty} a_n$ converges absolutely, you can rearrange the terms in the sum and the value _____
- If $\sum_{n=1}^{\infty} a_n$ converges conditionally,

Fact: Conditionally convergent sums can be rearranged to converge to